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LIGHT-RAIL TRANSIT: COST AND OUTPUT

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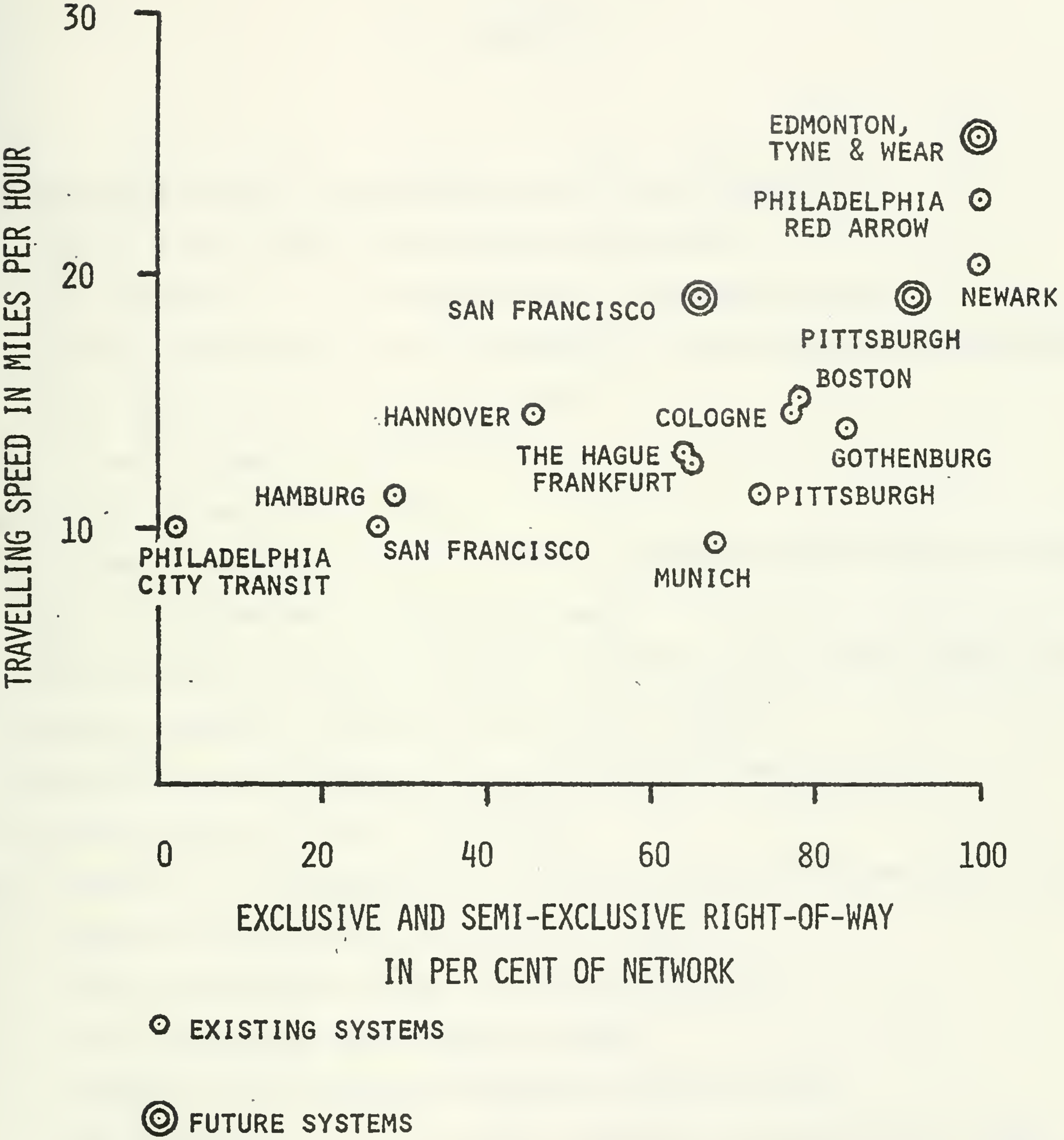
SUMMARY:

Growing concerns about environment, energy, and congestion demand a fresh look at mass transit, particularly nonpolluting, low-noise, nonoil-consuming, high-speed electric transit. The paper attempts to relate cost and output, both for light rail and the alternative transit mode, i. e. the express bus. For that purpose a simple algebraic framework is developed, tailored to data made available by the U. S. Department of Transportation. The volume of output is found at which, considering cost alone, a mass-transit authority would be indifferent between light rail and express buses.

Following a Continental European lead, rehabilitation of existing light-rail systems is currently taking place in Boston and San Francisco and is planned in Cleveland, Philadelphia, and Pittsburgh. New systems are under construction in Edmonton and Newcastle (Tyne & Wear), are about to be built in Buffalo and Calgary, and are being studied in several cities in the United States.

Why this new interest? Growing concerns about environment, energy, and traffic congestion are demanding a fresh look at mass transit, particularly nonpolluting, low-noise, nonoil-consuming, high-speed electric transit. Light-rail construction costs are moderate compared to heavy rail. Yet light-rail systems on exclusive and semi-exclusive rights-of-way may achieve quite satisfactory travelling speeds, as shown in Figure 1. High travelling speed is imperative, both in order to attract ridership and to reduce cost in dollars per line mile per year.

The present paper is concerned with supply rather than demand and will attempt to relate cost and output, both for light rail and the alternative transit mode, i. e. the express bus. The paper will develop a simple algebraic framework tailored to data derived from a study of alternative transit modes for the Pittsburgh South Hills corridor, prepared by De Leuw Cather and Company for the Port Authority of Allegheny County (1976) and made available in a more comprehensive study of the state of light-rail art prepared for the U. S. Department of Transportation, Urban Mass Transportation Administration (1976). The paper will use such data as input into its algebraic framework and find, first, numerical cost functions for light rail and express buses and, second, a specific volume of output at which, considering cost alone, a mass-transit authority would be indifferent between light rail and express buses. Thus the paper is an exercise in the interpretation rather than the collection of data.



DATA: DE LEUW, CATHER AND COMPANY (1976), 84. KILOMETERS CONVERTED TO MILES.

Figure 1

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I. NOTATION

A ≡ line-acquisition cost, dollars per line mile per year

a ≡ traffic cost, dollars per passenger-space mile

b ≡ time consumed by a train dwelling at a terminal, hours

β ≡ terminal dwelling time as a fraction of travelling time, pure number

C ≡ cost, dollars per line mile per year

d ≡ distance between terminals, miles

F ≡ number of vehicles in fleet

f ≡ frequency, average number of trains starting a round trip per hour

h ≡ hours of operation of system per year

j ≡ average number of vehicle hours of operation per fleet vehicle per year

k ≡ vehicle capacity, passenger spaces per vehicle

m ≡ operation and maintenance cost, dollars per vehicle mile

n ≡ average number of trains operating

ω_i ≡ fraction of h during which i-thirds of fleet is operating, i = 1, 2, 3

P ≡ line-acquisition price, dollars per line mile

p ≡ vehicle-acquisition price, dollars per vehicle

r ≡ rate of interest, pure number per year

s ≡ travelling speed between terminals, miles per hour

T ≡ time consumed by a train between starting two consecutive round trips, hours

t ≡ time consumed by a train travelling between terminals, hours

u ≡ useful life of line or vehicle, years

v ≡ average number of vehicles per train

W ≡ output per fleet vehicle, passenger-space miles in both directions per fleet vehicle per year

$X \equiv$ output per line mile, passenger spaces in both directions per year

$Y \equiv$ output of line, passenger-space miles in both directions per year

II. OUTPUT

1. Output of Line

Imagine a single line d miles long on which f trains are starting a round trip every hour for h hours per year. Each train consists of v vehicles, and each vehicle has k passenger spaces. The output of the line in passenger-space miles in both directions per year is then

(1) $$Y \equiv 2dfhkv$$

i. e. two <u>times</u> miles in one direction <u>times</u> trains per hour <u>times</u> hours per year <u>times</u> passenger spaces per vehicle <u>times</u> vehicles per train equalling passenger-space miles in both directions per year.

Define travelling speed between terminals as distance between terminals divided by time consumed between terminals

(2) $$s \equiv d/t$$

If T is the number of hours consumed by a train between starting two consecutive round trips, then 1/T is the number of times that train can start a round trip per hour. And n/T is the number of times all n trains can start a round trip per hour. But that is the same thing as traffic frequency:

(3) $$f \equiv n/T$$

The time consumed by a train between starting two consecutive round trips must include, first, twice the travelling time between terminals and, second, twice the terminal dwelling time:

(4) $$T \equiv 2(t + b)$$

Let terminal dwelling time be in direct proportion to travelling time between terminals:

(5) $$b = \beta t$$

where β is a parameter equalling, say, 1/10.

Insert (5) into (4), (4) into (3), (3) and (2) into (1) and find output of line

(6) $$Y = hknsv/(1 + \beta)$$

i. e. hours per year times passenger spaces per vehicle times average number of trains operating times miles per hour times average number of vehicles per train divided by a pure number, equalling passenger-space miles in both[1] directions per year.

2. *Output Per Fleet Vehicle*

Services cannot be stored but must be produced when demanded, and transit demand fluctuates over time. Consequently transit authorities must live with the fact that their full fleet is needed for only a fraction of the hours of operation of the system. We, too, must live with that fact and allow for it as follows. Define output per fleet vehicle as

(7) $$W \equiv Y/F = hknsv/[(1 + \beta)F]$$

i. e. passenger-space miles in both directions per fleet vehicle per year. Here n is average number of trains operating, and v is average number of vehicles per train, hence nv is average number of vehicles operating. That number must be less than or equal to the number of vehicles in the fleet:

(8) $$nv \leq F$$

A bus is a one-vehicle train: $v = 1$. Here nv may fall short of F because traffic frequency f falls short of its maximum. So it may in light-rail operations. But a light-rail train may be a multi-vehicle train: $v = 1, 2, 3$. Here nv may also fall short of F because number of vehicles per train v falls short of its maximum.

The average number of vehicles operating nv is a weighted average of the numbers of vehicles operating at different times. For simplicity's sake consider only three possible numbers of vehicles, i. e. $^1/_3$ of fleet, $^2/_3$ of fleet, and full fleet operating. Define ω_i as the fraction of hours of operation h during which i-thirds of the fleet is operating, where $i \equiv 1, 2, 3$ and $\omega_1 + \omega_2 + \omega_3 \equiv 1$. Spelled out, this means that

$$nv = {}^1/_3F \text{ for } \omega_1 h \text{ hours per year}$$
$$nv = {}^2/_3F \text{ for } \omega_2 h \text{ hours per year}$$
$$nv = F \text{ for } \omega_3 h \text{ hours per year}$$

In other words, ω_i are the weights of the weighted average of the numbers of vehicles operating at different times. That average is

(9) $$nv = ({}^1/_3\omega_1 + {}^2/_3\omega_2 + \omega_3)F$$

For example, if $\omega_1 = \omega_2 = \omega_3 = 1/3$ then $nv = 2/3 F$.

Now among other things, Eq. (7) contains the ratio

$$hnv/F \equiv j \tag{10}$$

i. e. hours per year <u>times</u> average number of trains operating <u>times</u> average number of vehicles per train <u>divided by</u> number of vehicles in fleet, or average number of vehicle hours of operation per fleet vehicle per year——a measure of vehicle utilization. Use (10) to write (7) as

$$W \equiv Y/F = jks/(1 + \beta) \tag{11}$$

i. e. average number of vehicle hours of operation per fleet vehicle per year <u>times</u> passenger spaces per vehicle <u>times</u> miles per hour <u>divided by</u> a pure number, or passenger-space miles in both directions per fleet vehicle per year——a measure of vehicle productivity needed in Sec. 6.

3. *Output Per Line Mile*

Output per line mile is defined as

(12) $$X \equiv Y/d = hknsv/[(1 + \beta)d]$$

i. e. passenger-space miles in both directions per year per mile or simply passenger spaces in both directions per year.

III. COST

4. *A Linear Cost Function*

As a first approximation let us estimate the linear cost function

(13) $$C = A + aX$$

where C is cost in dollars per line mile per year; A is line-acquisition cost in dollars per line mile per year; a is traffic cost in dollars per passenger-space mile; and X is output per line mile in passenger spaces in both directions per year.

Given the line with its guideway, track, stations, yards and shops, power collection and distribution, control and crossing protection, output per line mile may be expanded by expanding traffic. According to Eq. (11) at given vehicle hours of operation per fleet vehicle per year

j, given vehicle capacity k, given travelling speed s, and given dwelling-time fraction β, output per fleet vehicle $W = jks/(1 + \beta)$ will be given, and traffic can be expanded only by expanding the fleet F. As a first approximation, such expansion may be assumed to be possible at constant traffic cost a——up to an upper bound.

That upper bound is determined by technology plus safety considerations: At high-speed operation and automatic block signalling systems, train intervals ("headways") cannot be lower than 90 to 120 seconds. A 120-second headway means a train every other minute or a traffic frequency $f = 30$ trains per hour. At such a traffic frequency, what would output per line mile $X \equiv Y/d = 2fhkv$ be like?

In a light-rail operation let hours of operation per year be $h = 3900$; let vehicle capacity be $k = 115$ passenger spaces per vehicle; and let average number of vehicles per train be $v = 2$. Output per line mile will then be $X = 53.8$ million passenger spaces in both directions per year.[2]

5. *Line-Acquisition Cost*

Including costs of guideway, track, landscaping, stations, parking lots, rights of way, ice and snow control, yards and shops, power collection and distribution, control and crossing protection, engineering, administration, and contingencies, De Leuw, Cather and Company (1976), 273, estimated the price of a new line in dollars per line mile and found:

	Busway	Light rail
Line-acquisition price P, millions of dollars per line mile	8.61	11.05

Acquisition price P may be allocated among individual years of useful life by multiplying it by r + 1/u, where r is the rate of interest in a pure number per year, u is useful life of the line in years, and 1/u the depreciation coefficient in a pure number per year. Let r = 0.08 and u = 30, then r + 1/u = 0.1133 and

	Busway	Light rail
Line-acquisition cost $A \equiv (r + 1/u)P$, millions of dollars per line mile per year	0.98	1.25

6. *Traffic Cost: Vehicle Acquisition*

Traffic cost includes, first, vehicle-acquisition cost and, second, cost of operation and maintenance. We begin with vehicle-acquisition cost. The vehicle-acquisition price of a light-rail vehicle was estimated by De Leuw, Cather and Company (1976), 222, and the conventional 40-foot urban bus currently used by most U. S. mass transit operations is priced around $100,000. Thus

	Express bus	Light rail
Vehicle-acquisition price p, dollars per fleet vehicle	100,000	400,000

Acquisition price p may be allocated among individual years of useful life by multiplying it by r + 1/u, where r is the rate of interest in a pure number per year, u is useful life of vehicle in years, and 1/u the depreciation coefficient in a pure number per year. Let r = 0.08. County of San Diego (1975), 127 and 139, estimated useful life u to be 15 years for buses and 30 years for light-rail vehicles. Consequently r + 1/u = 0.14667 for buses and 0.11333 for light rail -vehicles, and

	Express bus	Light rail
Vehicle-acquisition cost (r + 1/u)p, dollars per fleet vehicle per year	14,667	45,332

But we don't want vehicle-acquisition cost in dollars per vehicle per year but in dollars per passenger-space mile. To find the latter we need Eq. (11).

On the right-hand side of (11) we find the quotient jks/(1 + β),

i. e. passenger-space miles in both directions per fleet vehicle per year. Let us find a plausible empirical value for it. Let a rather austere U. S. transit system operate for 15 hours a day 5 days a week 52 weeks a year. In that case h = 3900 hours per year. Assume as we did in Sec. 2 that $\omega_1 = \omega_2 = \omega_3 = {}^1/_3$. It follows from Eq. (9), then, that $nv = {}^2/_3F$: On an average, two-thirds of the fleet is operating. According to Eq. (10), vehicle hours of operation per fleet vehicle per year will then be j = 2600.[3] Let terminal dwelling time as a fraction of travelling time be $\beta = 1/10$. Define a passenger space as five square feet of vehicle floor space. Consult De Leuw, Cather and Company (1976), 275, on vehicle capacity k and travelling speed s and find:

	Express bus	Light rail
Vehicle use j, hours of operation per fleet vehicle per year	2,600	2,600
Vehicle capacity k, passenger spaces per vehicle	67	115
Travelling speed s, miles per hour	14	20
Terminal dwelling-time fraction β, pure number	0.10	0.10
Output per fleet vehicle jks/(1 + β), millions of passenger-space miles in both directions per fleet vehicle per year	2.22	5.44

Now divide vehicle-acquisition cost by output per fleet vehicle:

	Express bus	Light rail
Vehicle-acquisition cost $(r + 1/u)p$, dollars per fleet vehicle per year	14,667	45,332
Output per fleet vehicle $jks/(1 + \beta)$, millions of passenger-space miles in both directions per fleet vehicle per year	2.22	5.44
Former divided by latter: Vehicle-acquisition cost $(1 + \beta)(r + 1/u)p/jks$, dollars per passenger-space mile	0.0066	0.0083

In terms of dollars per passenger-space mile, then, a light-rail vehicle is merely 1.26 times as expensive as a bus. Let us summarize our dramatic whittling down of the rail-bus acquisition cost differential.

7. Vehicle Acquisition: Summary

On the face of it, a light-rail vehicle is four times as expensive as a bus: The vehicle-acquisition price p in dollars per vehicle of a light-rail vehicle is four times that of a bus. But such a statement considers neither useful life nor productivity. Because the light-rail vehicle has twice the useful life, its vehicle-acquisition cost $(r + 1/u)p$ in dollars per vehicle per year is only 3.09 times that of a bus. Furthermore, the light-rail vehicle is more productive: Its vehicle capacity k is 1.72 times higher, and its travelling speed s is 1.43 times higher. As a result, its vehicle-acquisition cost

$(1 + \beta)(r + 1/u)p/jks$ in dollars per passenger-space mile is only 1.26 times that of a bus. Considering both useful life and productivity, then, a light-rail vehicle is merely 1.26 times as expensive as a bus. But it <u>is</u> more expensive. The picture will be reversed once costs of operation and maintenance are considered.

8. *Traffic Cost: Operation and Maintenance*

De Leuw, Cather and Company (1976), 274, estimate costs of operation and maintenance in dollars[4] per vehicle mile. They are easily translated into dollars per passenger-space mile as follows:

	Express bus	Light rail
Operation and maintenance costs m, dollars per vehicle mile:		
Maintenance of rights-of-way	0.037	0.191
Energy	0.101	0.209
General and administration	0.628	0.532
Vehicle maintenance	0.258	0.216
Conducting transportation	0.852	0.650
Total, m, dollars per vehicle mile	1.876	1.798
Vehicle capacity k, passenger spaces per vehicle	67	115
Former divided by latter: Operation and maintenance costs m/k, dollars per passenger-space mile	0.0280	0.0156

Let us summarize the rail-bus cost differential for the various items of operation and maintenance cost.

9. *Operation and Maintenance: Summary*

The operation and maintenance cost of moving a light-rail vehicle one mile is 0.96 times that of moving a bus——practically the same. But the rail-bus cost differential differs widely among items: Maintenance of rights-of-way is 5.16 times as high; energy 2.07 times as high; general and administration 0.85 times as high; vehicle maintenance 0.84 times as high; and conducting transportation 0.76 times as high.

For the light-rail superiority in conducting transportation there are two reasons. First, higher travelling speed s: Let the money wage rate in dollars per hour be the same for bus and light rail. Then wages in dollars per vehicle mile will be lower for the vehicle travelling more miles per hour. The second reason is larger train size v. A bus is a one-vehicle train: $v = 1$. A light-rail train may be a multi-vehicle train: $v = 1, 2,$ or 3. The Pittsburgh South Hills project assumed a maximum $v = 3$. Now a three-vehicle hook-up saves no energy: The three -vehicle train consumes three times as much energy as the one-vehicle

train. But the hook-up does save labor: The two trains both need one driver. Consequently the driver's wages in dollars per vehicle mile of the three-vehicle train are only one-third of those of the one-vehicle train.

The operation and maintenance cost of moving a light-rail vehicle one mile was practically the same as that of moving a bus. But the operation and maintenance cost of moving a light-rail passenger space one mile is only 0.56 times that of moving a bus passenger space——little over one-half. The reason for this light-rail superiority is simply higher vehicle capacity: The light-rail vehicle carries 1.72 times as many passenger spaces as a bus.

In short, the light-rail superiority in operation and maintenance cost lies in, first, higher travelling speed s, second, larger train size v and, third, higher vehicle capacity k.

10. Overall Traffic Cost

The two components of traffic cost, i. e. vehicle-acquisition cost and operation and maintenance cost, have now been expressed in the same dimension, hence are additive. So we add them as follows:

	Express bus	Light rail
Vehicle-acquisition cost $(1 + \beta)(r + 1/u)p/jks$, dollars per passenger-space mile	0.0066	0.0083
Operation and maintenance costs m/k, dollars per passenger-space mile	0.0280	0.0156
Sum: Traffic cost $a \equiv (1 + \beta)(r + 1/u)p/jks + m/k$, dollars per passenger-space mile	0.0346	0.0239

The traffic cost of moving a light-rail passenger space one mile is only 0.69 times that of moving a bus passenger space——little more than two-thirds.

11. Linear Cost-Function Estimates

Write our linear cost function (13) for express buses and light rail

$$C = A_1 + a_1X \tag{14}$$

$$C = A_2 + a_2X, \tag{15}$$

respectively. Secs. 5 through 10 have determined the parameters:

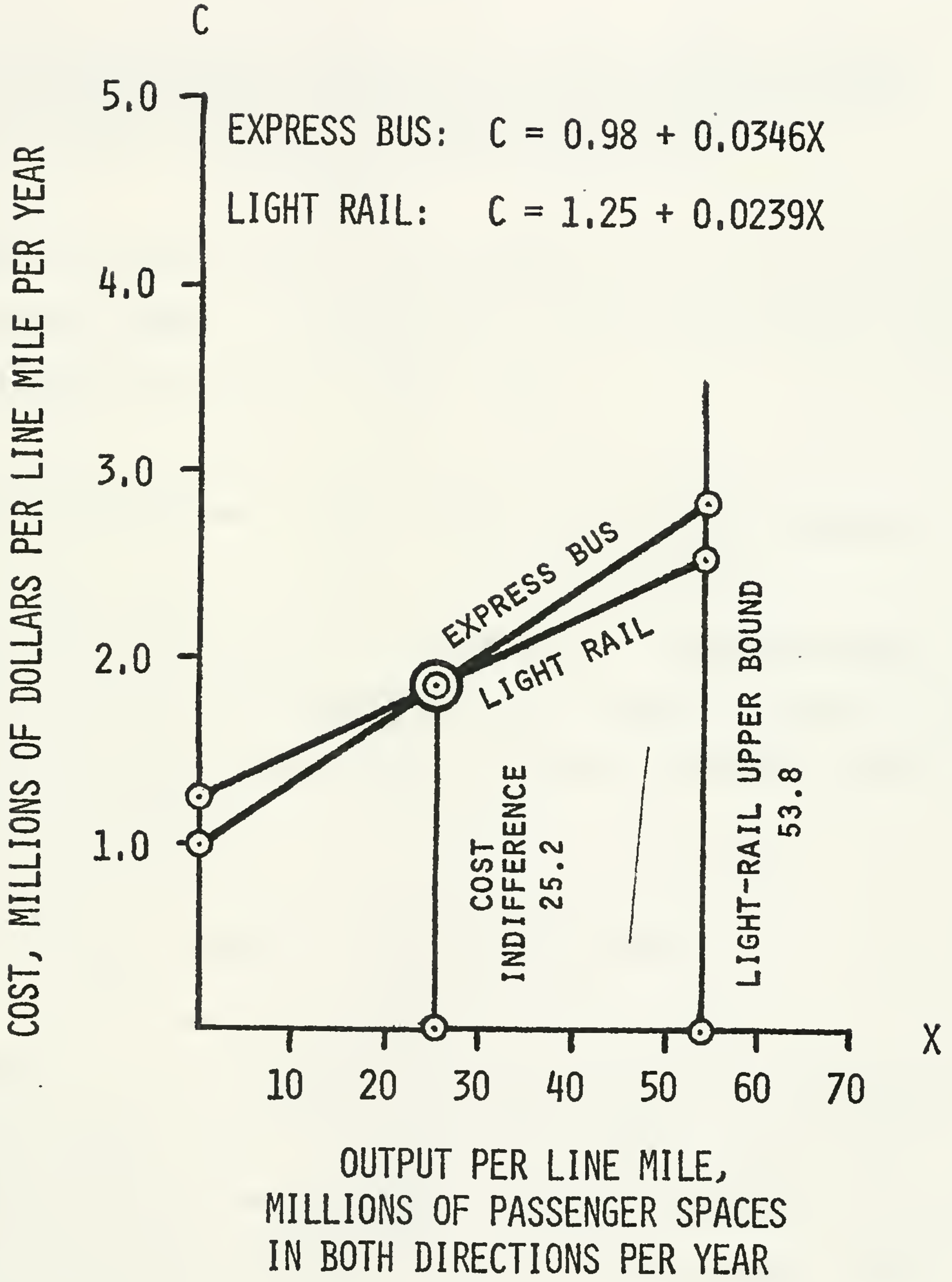

Figure 2

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A_1 = 0.98 millions of dollars per line mile per year
A_2 = 1.25 millions of dollars per line mile per year
a_1 = 0.0346 dollars per passenger-space mile
a_2 = 0.0239 dollars per passenger-space mile

So light rail has a higher line-acquisition cost but lower traffic cost than express buses. Consequently it will depend upon output which of the two modes has the lower cost.

12. Cost Indifference Between Express Buses and Light Rail

At precisely which output per line mile X, in passenger spaces in both directions per year, will express buses and light rail have the same cost C in dollars per line mile per year? Set the right-hand sides of (14) and (15) equal and find

$$X = (A_2 - A_1)/(a_1 - a_2) = 25.2 \text{ million} \qquad (16)$$

which is feasible, i. e. slightly less than one-half of the upper bound X = 53.8 million found in Sec. 4. Considering cost alone, then, a transit authority would be indifferent between express buses and light rail at an output per line mile equalling X = 25.2 million passenger spaces in both directions per year. Figure 2 finds the same result graphically by plotting the two cost functions and measuring the abscissa of their intersection point.

13. Other Measures of Traffic Density at Cost-Indifference Point

At our cost-indifference point $X = 25.2$ million passenger spaces in both directions per year, what would traffic frequency f be like? Take (1) and (12) together and express traffic frequency

$$f = X/(2hkv) \quad (17)$$

In our express-bus operation let hours of operation per year be $h = 3900$; let vehicle capacity be $k = 67$ passenger spaces per vehicle; and let number of vehicles per train be $v = 1$. According to (17) average frequency will then be $f = 48.2$ buses starting a round trip per hour——a bus starting every 1.24 minutes.

In our light-rail operation let hours of operation per year be $h = 3900$; let vehicle capacity be $k = 115$ passenger spaces per vehicle; and let average number of vehicles per train be $v = 2$. According to (17) average frequency will then be $f = 14.0$ trains starting a round trip per hour——a two-vehicle train starting every 4.29 minutes.

At our cost-indifference point, what would the number of passengers per year be like——as distinct from the number of passenger spaces per year? De Leuw, Cather and Company (1976), 274, offer data on cost per vehicle mile as well as per passenger mile. Dividing the

former by the latter we find the average number of passengers per vehicle operating to be 17.7 for express buses and 28.1 for light rail. This means an average space utilization of about one-fourth for both modes. Consequently our 25.2 million passenger spaces in both directions per year would be equivalent to about 6.3 million passengers in both directions per year or to about 1620 passengers in both directions per hour. County of San Diego (1975), 89, hints at a cost -indifference point at 1500 to 2000 passengers per hour. Beyond that, "light rail could in many cases provide a higher level of service at lower cost".

14. The Sensitivity of the Cost-Indifference Point

The cost-indifference point is sensitive to changes in the four parameters A_1, A_2, a_1, and a_2. Changes in the slopes a_1 and a_2 are particularly interesting. Assume express buses to be diesel-powered. Assume light-rail vehicles to be propelled by electricity generated by coal-fired, hydroelectric, or nuclear power plants. A higher relative price of oil would then raise the slope a_1 but leave the slope a_2 unaffected. The effect upon the cost-indifference point would be described numerically as follows. In (16) take the partial derivative of X with respect to a_1. Multiply by the ratio a_1/X and arrive

at the elasticity of X with respect to a_1:

$$(18) \qquad \frac{\partial X}{\partial a_1}\,\frac{a_1}{X} = -\,\frac{A_2 - A_1}{(a_1 - a_2)^2}\,\frac{a_1}{X} = -\,\frac{a_1}{a_1 - a_2} = -\,3.23$$

which is valid in the immediate neighborhood of the cost-indifference point. The elasticity (18) is negative and says that raising the slope a_1 by one per cent will reduce X by 3.23 per cent, in other words move the cost-indifference point 3.23 per cent to the left.

Next, let light-rail vehicles be propelled at first by electricity generated by oil-fired power plants. A conversion to presumably cheaper nuclear power would then reduce the slope a_2 but leave the slope a_1 unaffected. The effect upon the cost-indifference point would be described numerically as follows. In (16) take the partial derivative of X with respect to a_2. Multiply by the ratio a_2/X and arrive at the elasticity of X with respect to a_2:

$$(19) \qquad \frac{\partial X}{\partial a_2}\,\frac{a_2}{X} = \frac{A_2 - A_1}{(a_1 - a_2)^2}\,\frac{a_2}{X} = \frac{a_2}{a_1 - a_2} = 2.23$$

which is valid in the immediate neighborhood of the cost-indifference point. The elasticity (19) is positive and says that reducing the slope a_2 by one per cent will reduce X by 2.23 per cent, in other words move the cost-indifference point 2.23 per cent to the left.

15. Beyond the Cost-Indifference Point

Beyond cost considerations lie such factors as uncertainty and environment.

First uncertainty. The useful lives of a light-rail line and a busway were assumed to be the same. But the light-rail line -acquisition price P in dollars per line mile was 1.28 times that of the busway. A light-rail line is simply a larger investment than a busway. The useful life u of a light-rail vehicle was twice that of a bus, hence the depreciation coefficient 1/u was only one-half. Thus light-rail vehicles are a longer investment than buses. On a per-mile basis, then, a light-rail system is both a larger and a longer investment than an express-bus system. And in an uncertain world, a larger and longer investment may seem less attractive than a smaller and shorter one——even if the latter may have a higher cost in dollars per line mile per year! In such cases an express-bus system may be preferred even in ranges lying to the <u>right</u> of our cost-indifference point.

Second environment. Lower tolerance thresholds of the noise, carbon monoxide, hydrocarbons, and nitrogen oxide generated by

express buses, cf. De Leuw, Cather and Company (1976), 207-213, will affect neither our cost functions nor their intersection point. But lower tolerance thresholds may make a light-rail system preferred even in ranges lying to the left of our cost-indifference point.

IV. CONCLUSION

The present paper might be called a case study: Its empirical base is the Pittsburgh South Hills corridor project, and conditions elsewhere may be quite different. With this reservation the conclusion of the paper is the following.

We have found cost indifference between express buses and light rail at an output per line mile of 25.2 million passenger spaces in both directions per year——less than half the upper bound, 53.8 million passenger spaces in both directions per year. At outputs between 25.2 and 53.8 million, then, light rail is cheaper than express buses. But the cost superiority is moderate: Even at its upper bound, light rail still costs 89 per cent of express buses and saves merely 0.31

million dollars per line mile per year.

This means that, first, at large outputs cost considerations certainly do favor light rail and, second, at such outputs the <u>other</u> advantages of light rail may well be more decisive than its cost advantage. At such outputs, bus slowness, congestion, noise and pollution may long since have reached their tolerance thresholds.

F O O T N O T E S

*For discussion in a seminar at the Copenhagen School of Business in September 1977, the author is indebted to Mr. K. N. Andersen, President of the Greater Copenhagen Transit Authority (Hovedstadsområdets Trafikselskab). For demonstration of a modern light-rail system and interview, also in September 1977, the author is indebted to Mr. Ragnar Domstad, Vice-President in charge of planning at the Gothenburg Transport Authority (Göteborgs Spårvägar). For critical reading of an earlier version of the manuscript the author is indebted to his colleague John F. Due.

[1]The present paper is concerned with the cost-output relationship. Output is measured in passenger-space miles per year. Such output is produced, and cost is incurred in producing it, whether the passenger spaces happen to be occupied or not, and whether the vehicle is travelling in one or the other direction. Throughout the present paper, therefore, output is measured in both directions.

Engineers concerned with maximum output often measure that output in one direction only, cf. De Leuw, Cather and Company (1976), 275 and possibly 204.

[2]Engineers concerned with maximum output are interested in occasionally higher hourly densities based on a minimum 90-second headway and a maximum number of vehicles per train $v = 3$. But throughout the present paper, output as well as its upper bound are yearly averages.

[3]Such a value is, perhaps, close to U. S. practice. Implicitly De Leuw, Cather and Company (1976), diagram on Page 240, show the following values of our j:

Pittsburgh	1760
Newark	2500
New Orleans	3750
Basel, Bremen, Nuremberg	4510

Western European central business districts typically offer more attractions evenings and weekends than U. S. cities do. New Orleans is an exception with its unique French Quarter and almost European j.

[4]We follow De Leuw, Cather and Company in measuring line and vehicle acquisition cost in 1975 dollars but operation and maintenance costs in 1985 dollars. Interest and depreciation on line and vehicles should indeed be calculated on historical cost of acquisition, and operation and maintenance costs should indeed be those of a typical year of operation, say 1985, during useful life.

LITERATURE

COUNTY OF SAN DIEGO, Public Works Agency, Department of Transportation, *Investigation of a Light Rail System for the San Diego Region*, San Diego, California, 1975.

DE LEUW, CATHER AND COMPANY, *Light Rail Transit: State of the Art Review*, prepared for the U. S. Department of Transportation, Urban Mass Transportation Administration, distributed by the National Technical Information Service, U. S. Department of Commerce, Springfield, Virginia, 1976.

PORT AUTHORITY OF ALLEGHENY COUNTY, *Comparative Analysis Study of Alternative Transit Systems: South Hills Corridor*, Allegheny County, Pennsylvania, 1976.

END OF MANUSCRIPT

2 photos of diagrams enclosed.